Matrices

Question:

In a remote country town each adult is either *currently attending* a gym or *not currently attending* a gym. A person studying gym attendance in this town constructs a model for how attendance might change. It assumes that at the end of a given month,

– 30% of those attending a gym continues to attend in the next month, while 70% stopped attending.

– 70% of those *not* attending a gym change, and attend a gym in the next month, while 30% continue to not attend.

The matrix $A = \begin{bmatrix} 0.3 & 0.7 \\ 0.7 & 0.3 \end{bmatrix}$ can be used to represent this model.

Suppose at the start of January 2011 there will be 27000 adults attending a gym and 54000 adults not attending a gym.

Let $S_o = \begin{bmatrix} 27000 \\ 54000 \end{bmatrix}$

a) i) Write down a matrix expression that can be used to calculate the number of adults attending and not attending at the start of April 2011.

ii) Calculate number of adults attending and not attending at the start of January 2012.

Consider the following two different models for change in gym attendance.

– Model A $A = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}$

– Model B $A = \begin{bmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{bmatrix}$

b) For each model, find A^{20}

Now let $A = \begin{bmatrix} x & 1-x \\ 1-x & x \end{bmatrix}$

c) Based on your answers to part a) ii) and part b), make a conjecture about An when $n \to \infty$ and $0 < x < 1$, and provide an answer with respect to gym attendees

(7 marks)

Step	Method/ Hint	Answer	Marks allocation (where applicable)
PART (a)(i) of the question:			
	- Identify identity matrices (columns add to 1) - Identify that April is 3 transitions from January (FEB, MAR, APR) - Write the rule for transition matrices with 3 transitions	$S_3 = A^3 S_o$	1 mark

Step	Method/ Hint	Answer	Marks allocation (where applicable)
PART (a)(ii) of the question:			
	- Identify that January 2012 is 12 transitions - Complete the matrix multiplication for the rule $S_{12} = A^{12} S_o$ $S_{12} = \begin{bmatrix} 0.3 & 0.7 \\ 0.7 & 0.3 \end{bmatrix}^{12} \times \begin{bmatrix} 27000 \\ 54000 \end{bmatrix} \begin{array}{l} \boxed{\text{Attending}} \\ \boxed{\text{Not Attending}} \end{array}$ **NOTE :** - 2x2x2x1 - can be multiplied as the number of columns of the first matrix is equal to the number of rows in the second matrix (the two internal numbers of the orders are the same) - the answer will be a 2x1 matrix (the number of rows of the first matrix and the number of columns in the second matrix) - Answers must be given in whole numbers, as people can be in part - Both answers must total original adults in attendance (81 000 in this question)	*USE CALCULATOR TO EVALUATE* $S_{12} = \begin{bmatrix} 0.3 & 0.7 \\ 0.7 & 0.3 \end{bmatrix}^{12} \times \begin{bmatrix} 27000 \\ 54000 \end{bmatrix}$ $S_{12} = \begin{bmatrix} 40499.8 \\ 40500.2 \end{bmatrix} \begin{array}{l} \boxed{\text{Attending}} \\ \boxed{\text{Not Attending}} \end{array}$ **Answer:** According to this model there will be approximately 40500 adults attending a gym in January 2012, and the same number not attending. **NOTE** - attending + not attending = constant (ie. the total must equal 81 000 adults)	1 mark

Step	Method/ Hint	Answer	Marks allocation (where applicable)
PART (b) of the question:			
	Use Calculator to find A^{20} for model A and model B	Model A → $\begin{bmatrix}0.4 & 0.6\\0.6 & 0.4\end{bmatrix}^{20}$ = $\begin{bmatrix}0.5 & 0.5\\0.5 & 0.5\end{bmatrix}$ Model B→ $\begin{bmatrix}0.1 & 0.9\\0.9 & 0.1\end{bmatrix}^{20}$ = $\begin{bmatrix}0.506 & 0.494\\0.494 & 0.506\end{bmatrix}$	1 mark 1 mark (2 marks total)

Step	Method/ Hint	Answer	Marks allocation (where applicable)
PART (c) of the question:			
	Identify that is asking for the steady state values (when $n \to \infty$) - Take each model to a high power and note the values for each - Once steady values are identified (ie. model A matches values of model B), place 'n' in the position of the high power - Identify what this informs about the number of attendees to a gym	Model A → $\begin{bmatrix}0.4 & 0.6\\0.6 & 0.4\end{bmatrix}^{50}$ = $\begin{bmatrix}0.5 & 0.5\\0.5 & 0.5\end{bmatrix}$ Model B→ $\begin{bmatrix}0.1 & 0.9\\0.9 & 0.1\end{bmatrix}^{50}$ = $\begin{bmatrix}0.5 & 0.5\\0.5 & 0.5\end{bmatrix}$ For either model A^n = $\begin{bmatrix}0.5 & 0.5\\0.5 & 0.5\end{bmatrix}$ Therefore, in the long term, 50% of the population will attend the gym and 50% will not	1 mark 2marks (3 marks total)