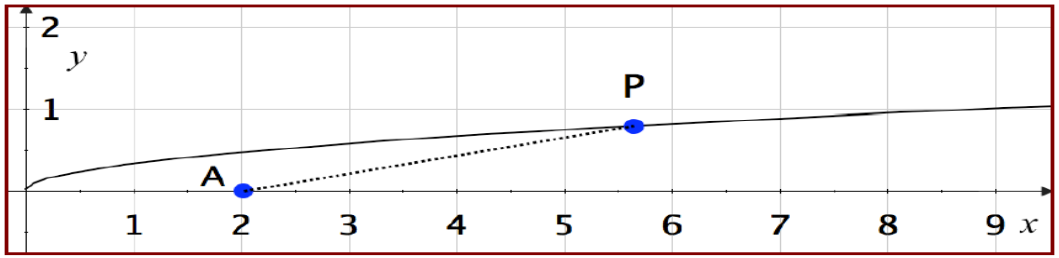
Application of Differentiation

Question: (10marks)

Suppose that a *moving body 'P'* orbits around a *stationary body 'A'* on a particular path.
Below is a simple model of this situation in which A is the fixed point (2, 0), P is the moving point (x, y) and where the path of the orbit is described by $y = \frac{1}{3}\sqrt{x}$ for $x \geq 0$.

Let D be the straight-line distance between A and P measured in units.



a) Find D when $x = 4$.

b) Show that $D = \sqrt{x^2 - \frac{35}{9}x + 4}$ $x \geq 0$.

c) Find the position of P for which D is a minimum. Give the co-ordinates of P as exact values.

Step	Method/ Hint	Answer	Marks allocation (where applicable)
Step 1	a) To find D when x= 4, First, find the vertical height of the point P from the x-axis - Identify that P is a moving point - Substitute x = 4 into the equation, of 'P', to find the vertical height of the point P	$y = \frac{1}{3}\sqrt{4}$ $y = \frac{1}{3}.2$ $y = \frac{2}{3}$	1 mark

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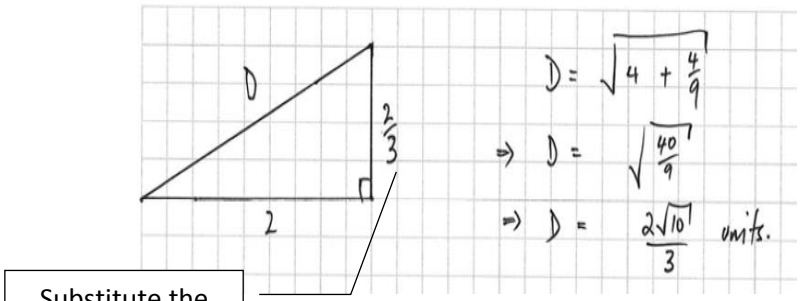
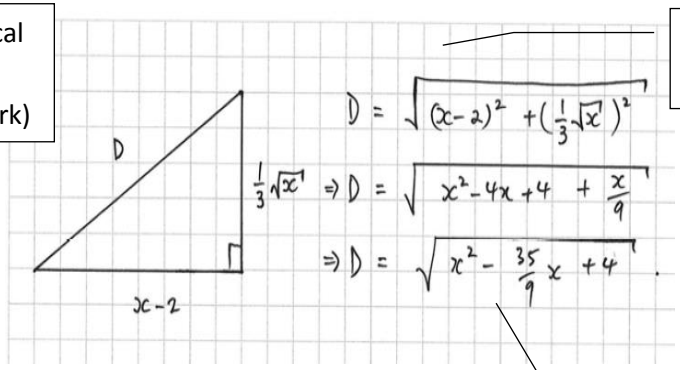
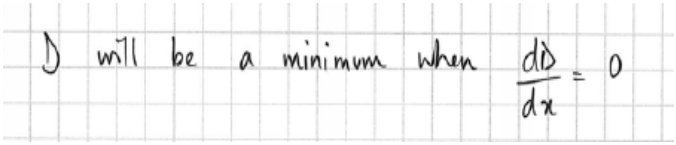
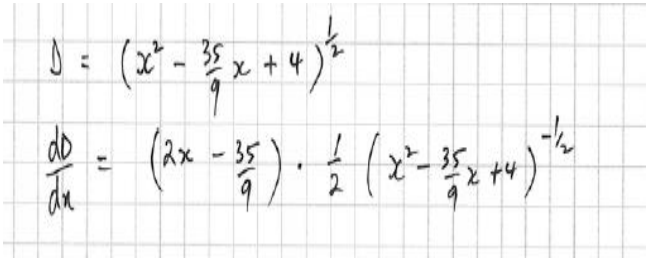
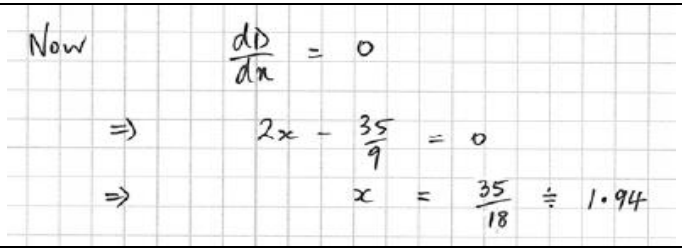
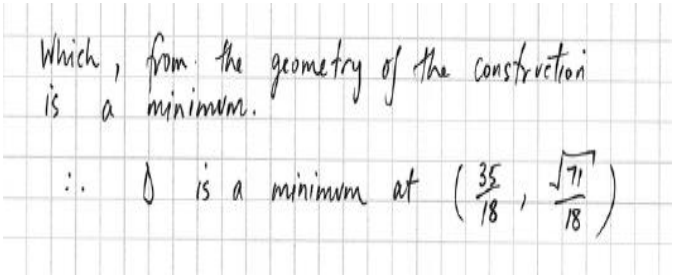
Step 1	- Create a right angle triangle and find the length of the hypotenuse, which establishes the straight line distance (D) from 'A' to 'P'.	 Substitute the vertical height found earlier	1 mark
Step 2	(b) To show that $D = \sqrt{x^2 - \frac{35}{9}x + 4}$,$x \geq 0$ - Identify that P is point (x,y) and can be described by Horizontally → x-2 Vertically → $\frac{1}{3}\sqrt{x}$ - Substitute the vertical and horizontal expressions into Pythagoras' Theorem to find the hypotenuse $D^2 = Hor^2 + Ver^2$ $D^2 = (x-2)^2 + (\frac{1}{3}\sqrt{x})^2$ - Expand and Simplify	Identify the vertical and horizontal expressions (1 mark)  Use Pythagoras' Theorem (1 mark) Expand and Simplify (1 mark)	3 marks

Step 3	c) To find the position of P for which D is a minimum and give the co-ordinates of P as exact values: - Identify that D will be a minimum when the Gradient Function is equal to zero		(5marks) - 1 mark
Step 4	- Find the derivative of the function • Change the $\sqrt{\ }$ into $(\ )^{\frac{1}{2}}$ • Use the Chain Rule to find the derivative of $(x^2 - \frac{35}{9}x + 4)^{\frac{1}{2}}$ **CHAIN RULE** $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ *Where* $u = x^2 - \frac{35}{9}x + 4$ $\quad \frac{du}{dx} = 2x - \frac{35}{9}$ $x = u^{\frac{1}{2}}$ $\quad \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ *then substitute 'u' back into equation*		- 1 mark

Step 5	- Identify the linear bracket, and make it equal to zero (Ensure all results are left in exact values)		- 1 mark
Step 6	- As noted, from the geometry of the construction of the diagram this is a minimum and calculate the y value at this point (again in exact values) - Calculate by substituting back into the original equation $D = \sqrt{(x^2 - \frac{35}{9}x + 4)}$ where x = $\frac{35}{18}$ $D = \sqrt{(\frac{35^2}{18^2} - \frac{35}{9}.\frac{35}{18} + 4)}$ $D = \sqrt{(\frac{1225}{324} - \frac{1225}{162} + 4)}$ $D = \sqrt{(\frac{1225}{324} - \frac{2450}{324} + \frac{1296}{324})}$ $D = \sqrt{(\frac{71}{324})}$ $D = \frac{\sqrt{(71)}}{18}$		-1 mark for substituting back into original equation to find D - 1 mark for final result indicating point

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