Number Patterns

Question:

The purchase and installation of a basic heating system with **five** outlets costs $3500. Each additional outlet costs an extra $80.

a. Determine the cost of installing a heating system with **eight** outlets.

b. A customer has $4400 to spend on a heating system and outlets. Determine the greatest number of outlets that can be bought with this heating system.

Australian Heating recommends that a house with 20 squares of living area should have 12 heating outlets.

c. Using this recommended ratio, determine the **cost** of installing a heating system for a house having 35 squares of living area.

Step	Method/ Hint	Answer	**Marks allocation** (where applicable)
PART (a) of the question:			
Step 1	Identify: It is an **Arithmetic sequence** The first term is 3500 for 5 (outlets) The common difference is an increase of 80 (dollars) **Option 1: Count on the sequence** Start with the first term t_1 for 5 outlets and add 80 until you reach 8 outlets.	5 outlets, 6 outlets, 7 outlets, 8 outlets **3500, 3580, 3660, 3740** t_1 t_2 t_3 t_4	
Step 2	State the answer for the cost of installing 8 answers.	Therefore the cost of installing a heating system with eight outlets is $3740	1 mark

OR Step 1	**Option 2: Use the formula for finding the term in an arithmetic sequence** Identify: - First term (a) - Common difference (d) - Number of terms (n) from 5 outlets to 8	a = $3500 d = $80 n = 4	
Step 2	State the formula needed: **$t_n = a + (n-1)d$** Substitute the values of **a, d** and **n** and evaluate	**$t_n = a + (n-1)d$** $t_4 = 3500 + (4-1) \times 80$ $= 3500 + 3 \times 80$ $= 3500 + 240$ **$= \$3740$**	
Step 3	State the answer for the cost of installing 8 answers.	Therefore the cost of installing a heating system with eight outlets is $3740	Or 1 mark

Step	**Method/ Hint**	**Answer**	**Marks allocation** (where applicable)
PART (b) of the question:			
Step 1	**Option 1: Count on the sequence** Continue the number pattern until the value 4400 is reached by adding 80 each time		1 mark (working)

No of outlets	5	6	7	8	9	10	11	12	13	14	15	16	17
Cost	3500	3580	3660	3740	3820	3900	3980	4060	4140	4220	4300	4380	4460

Step 2	**NOTE:** customer has $4400 so value cannot exceed 4440. The cost of installing 16 outlets is $4380 and 17 outlets is $4460, therefore needs to be one less	The greatest number of outlets that can be bought with this heating system is 16 outlets without exceeding $4400	1 mark (correct answer)
Step 2	**Option 2: Use the formula for finding the number of terms t_n in an arithmetic sequence** t_n = $4400 a = $3500 d = $80 Need to find n, the number of terms required	**$t_n = a + (n-1)d$** 4400 = 3500 + (n-1) x 80 900 = (n-1) x 80 900/80 = (n-1) 11.25 = n-1 n = 12.25 OR using **SOLVE** on calculator: Solve (4400 = 3500 + (n-1) x 80, n) ENTER n = 12.25	1 mark
Step 2 cont.	Once n is found the original 4 outlets must be added as the first number is outlet 5	12.25 +4 = 16.25 however 0.25 of an outlet cannot be used and therefore rounding down, ANSWER is 16 outlets The greatest number of outlets that can be bought with this heating system is 16 outlets without exceeding $4400	1 mark Total 2 marks

Step	Method/ Hint	Answer	Marks allocation (where applicable)
PART (c) of the question:			
Step 1	Identify the ratio required	20 squares of living : 12 outlets	
Step 2	Use the ratio to identify how many outlets are required in a home 35 square meters. Use 'n' to represent the unknown number of outlets	35 square of living: 'n' outlets	
Step 3	Form the proportion	$\frac{20}{35} = \frac{12}{n}$	1 mark
Step 4	Use cross multiplication to solve the equations	20n = 12 x 35 20n = 420 n = 420/20 n = 21	
Step 5	Take away 4 outlets from the answer calculated, as the initial cost of installation starts from 5 outlets	n = 21 - 4 n = 17	
Step 6	Substitute the remaining number of outlets into your expression or continue the number pattern t_n = ? n=17 a=3500 d=80	**t_n = a + (n-1)d** t_n= 3500 + (17-1)80 t_n= 3500 + 16 x 80 t_n= 3500 + 1280 t_n= 4780 The cost of installing a heating system for a house having 35 squares of living area is $4780 Or continuing the number pattern….. 5 outlets - 3500, 3580, 3660, 3740, 3820, 3900, 3980, 4060, 4140, 4220, 4300, 4380, 4460, 4540, 4620, 4700, 4780 – 21 outlets	1 mark Total 2 marks

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Best Practice #Unit 1/2 – General Maths